Engineering Notes

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Minimizing Mission Risk in Fuel-Constrained Unmanned Aerial Vehicle Path Planning

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I. Introduction

ONE of the main reasons for using an unmanned aerial vehicle (UAV) instead of a manned aircraft is when the mission that needs to be carried out is very dangerous. In such cases, we believe that the risk of losing the vehicle should be made an explicit and important part of the mission planning. The mentioned risks can often be estimated on a small scale, using detailed simulation models, but the tools to propagate such estimates to the overall mission-planning level has been missing. In this paper, we propose one way to take such local risk estimates into account in the mission planning and, at the same time, make the tradeoffs against other path properties, such as fuel or time constraints, explicitly. First, we show how to rewrite the accumulated risk estimate into a form that matches the formulation of the so-called weight-constrained shortest-path problem (WCSPP). Then we approximately solve it using a series of regular shortest-path problems (SPP). These are, in turn, easily solved by standard algorithms such as Dijkstra's as described in [1] or D* [2].

UAV path planning has been extensively studied in the literature [3–10]. However, to the best of our knowledge, only Chaudhry et al. [9] give the operator an explicit estimate of the risk associated with the proposed mission path. Furthermore, only Zabarankin et al. [11] enable the user to clearly state constraints in terms of available fuel or time. In contrast to both Chaudhry et al. [9] and Zabarankin et al. [11], who focus on the interactions of the UAV with a single surface-to-air missile (SAM) battery, the proposed approach enables planning of missions dealing with multiple threats, in the same way as done in the papers by Beard et al. [3] and others [4,7,10].

The organization of this paper is as follows. In Sec. II, we state the proposed risk-vs-fuel path-planning problem as well as two other path-planning problems from the literature. We then show how the proposed problem can be rewritten and approximately solved in Sec. III. A detailed example illustrating the approach is presented in Sec. IV, and the paper is concluded in Sec. V.

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II. Problem Formulations

In this section, we will define three problems: the SPP, the WCSPP, and the risk-vs-fuel-path problem (RFPP). These three problems and the corresponding acronyms will be used throughout the paper. Before defining the three problems, we briefly review some graph notation.

Definition 1: A graph $G=(V,E)$ is a collection of vertices $V=\{1\dots n\}$ and edges

$$E\subset V\times V=\{(i,j),i,j\in V\}$$

A path P is a sequence of vertices $P=(v_1,v_2,\dots,v_m)$ such that the set of consecutive pairs

$$E(P)=\{(v_i,v_{i+1}),i=1,\dots,m-1\}$$

is a subset of E. Each edge in a graph can furthermore be assigned weights $w_{ij}\in\mathbb{R}_+$ and costs $c_{ij}\in\mathbb{R}_+$. We are now ready to state a general SPP.

Problem 2.1 (SPP): Given a graph $G=(V,E)$, costs $c_{ij}\in\mathbb{R}_+$, and start and destination vertices s and d, the SPP is defined as follows:

$$\min_P \Sigma_{(i,j)\in E(P)}c_{ij} \qquad \text{subject to } s,d\in P \tag{1}$$

That is, find the path P from s to d such that the sum of costs c_{ij} is minimized.

The weight-constrained version of the SPP is similarly defined [11]:

Problem 2.2 (WCSPP): Given a graph $G=(V,E)$, weights $w_{ij}\in\mathbb{R}_+$, costs $c_{ij}\in\mathbb{R}_+$, and start and destination vertices s and d, the WCSPP is defined as follows:

$$\min_P \Sigma_{(i,j)E(P)}c_{ij} \qquad \text{subject to } \Sigma_{(i,j)E(P)}w_{ij}\le W \qquad s,d\in P \tag{2}$$

That is, find the path P from s to d such that the sum of costs c_{ij} is minimized and the sum of weights w_{ij} is kept below the bound W.

At this point, we note that the WCSPP is, in fact, an NP-hard (nondeterministic polynomial time hard) problem [12], which means that large problem instances are, in practice, very hard to solve to optimality in reasonable time. Having reviewed the SPP and the WCSPP, we now define the RFPP and discuss its properties in two remarks.

Problem 2.3 (RFPP): Given a graph $G=(V,E)$ with weights $w_{ij}\in\mathbb{R}_+$ corresponding to the amount of fuel needed to traverse the edge between nodes i and j and costs $R_{ij}\in[0,1]$ corresponding to the risk of losing the vehicle when traversing the edge between nodes i and j, and start and destination vertices s and d, the RFPP is defined as follows:

$$\max_P \Pi_{(i,j)E(P)}(1-R_{ij}) \quad \text{subject to } \Sigma_{(i,j)E(P)}w_{ij}\le W \qquad s,d\in P \tag{3}$$

That is, find the path that minimizes the accumulated risk of losing the vehicle, with the required amount of fuel being less than W.

Remark 1: Note that the UAV survival probability for the whole mission only equals $\Pi_{(i,j)\in E(P)}(1-R_{ij})$ if the path segment risks R_{ij} are uncorrelated. This is obviously a quite strong assumption.

However, we believe that it still makes more sense than the options of summing threat distances, radar exposure, or other measures of UAV risk. Furthermore, the choice of graph G and corresponding path segments can be made to increase the validity of this assumption by, for example, using a Voronoi graph on which segment lengths are on the same order of magnitude as the threat regions.

Remark 2: Throughout this paper we are using fuel as the main constraint W. However, other path properties such as distance or time of traversal can just as easily be used instead of fuel.

III. Proposed Solution

In this section, we will see how to solve the RFPP by rewriting it as a WCSPP, which can in turn be solved approximately using a series of SPPs.

A. Writing the RFPP as a WCSPP

Note that the fuel constraint and path criteria of the RFPP maps nicely into a WCSPP. The objective function is, however, a product in the RFPP, reflecting the combined probability of surviving all the path segments, and not a sum of costs as in the WCSPP. In many papers, such as Beard et al. [3] and Zabarankin et al. [11], overall risk is not minimized. Instead, the objective functions capture a sum of kill probabilities, a sum of inverse squared threat distances, or a sum of some other measure of problems. Here, however, we can find the solution that minimizes the actual risk estimate by an elaborate choice of edge costs described in the following Lemma.

Lemma: Let $c_{ij} = \log(1/(1 - R_{ij}))$. Then the RFPP (problem 2.3) has the same solution as the WCSPP (problem 2.2).

Proof: Because the constraints are the same in problems 2.3 and 2.2, we only need to show that the objective function of problem 2.3, that is, $\max \Pi_{e_i \in \text{path}}(1 - R_{ij})$, can be manipulated into something of the form $\min \Sigma_{e_i \in \text{path}} c_{ij}$ without changing the corresponding optimal solution. Now, using $\Leftrightarrow$ to denote that two optimization problems have the same solution, we have

$$\max \Pi_{e_i \in \text{path}}(1 - R_{ij}) \Leftrightarrow \max \ \log(\Pi_{e_i \in \text{path}}(1 - R_{ij}))$$

because log is a strictly increasing function and can therefore be applied to any objective function in an optimization problem without changing the corresponding optimal solution. Using $\log \Pi = \Sigma \log$, we get

$$\max \ \log(\Pi_{e_i \in \text{path}}(1 - R_{ij})) \Leftrightarrow \max \Sigma_{e_i \in \text{path}} \log(1 - R_{ij})$$

Furthermore, because a maximization problem can be turned into a minimization problem by multiplying the objective function by -1, we have

$$\max \Sigma_{e_i \in \text{path}} \log(1 - R_{ij}) \Leftrightarrow \min \Sigma_{e_i \in \text{path}} - \log(1 - R_{ij})$$

Finally, applying $-\log x = \log(1/x)$, we get

$$\min \Sigma_{e_i \in \text{path}} - \log(1 - R_{ij}) \Leftrightarrow \min \Sigma_{e_i \in \text{path}} \log(1 - R_{ij})^{-1}$$

which is exactly the proposed c_{ij}.

B. Solving the WCSPP

Zabarankin et al. [11] modified a version of the label-setting algorithm [12]. In this paper, we use bisection search and a standard shortest-path algorithm, such as the Dijkstra algorithm in [1], to get a good feasible solution and an upper bound on the gap to optimum. The details of this approximation scheme can be found in our previous work [13]. Knowing how to transform the RFPP into a WCSPP that can be approximately solved by a series of SPP or a label-setting algorithm [12], we go on to illustrate the approach with an example.

IV. Example Problem

In this section, we will find minimum risk paths for different fuel constraint levels in the example scenario depicted in Fig. 1. The task

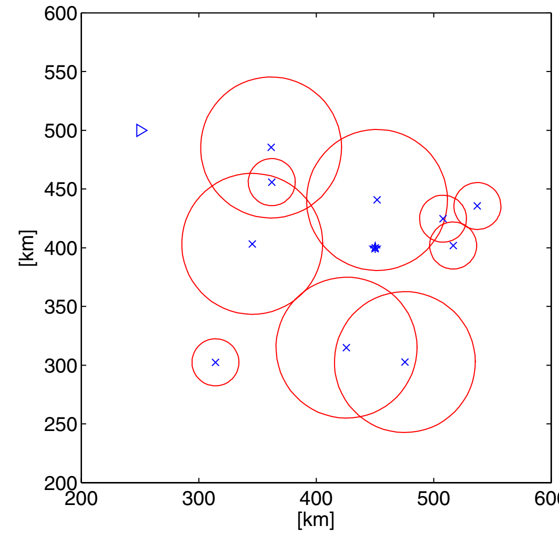

Fig. 1 Setup of a path-planning problem; threat regions are illustrated by circles.

of the planner is to find a path from the start position at (250,500) to the target position at (450,400). As can be seen in Fig. 1, the target is located within a threat area. This is fairly reasonable, because the target is supposed to be worth flying to and thus probably worth protecting. It should also be noted that this case immediately disqualifies approaches in which all threat areas must be avoided. To formulate a RFPP we need to estimate the risks of flying in different parts of the mission theater. Finding a risk estimate for a given flight-path segment is a task that in itself can be made arbitrarily complex, as noted in Remark 3.

Remark 3: We acknowledge that risks are very hard to estimate. However, using intelligence information and high-fidelity simulation models of threat systems and electronic warfare components, it is possible to make coarse estimates, similar to those found in the paper by Chaudhry et al. [9]. Furthermore, incorporating such results in a UAV path planner is one way to make sure that the knowledge of the subject matter experts is indeed put to use by the men and women making tactical decisions in the field.

We do not go into details about risk estimation here, but note that a threat-level map, indicating the risk of flying a path segment of a given length, could look like the one depicted in Fig. 2.

The next step is to choose the form of the graph G in which to plan the path. There are many options, including Voronoi grids [3],

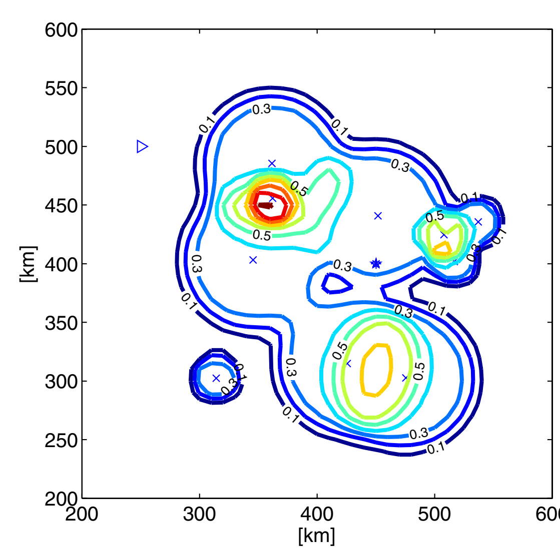

Fig. 2 Estimated probability of losing the UAV when flying a path segment.

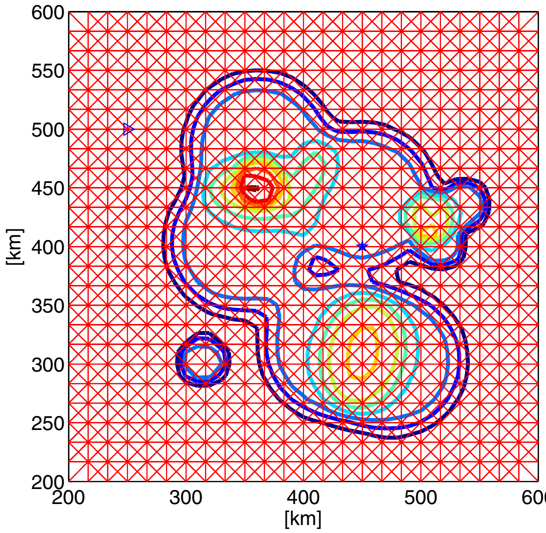

Fig. 3 One possible choice of the graph *G*, drawn on top of the risk estimates.

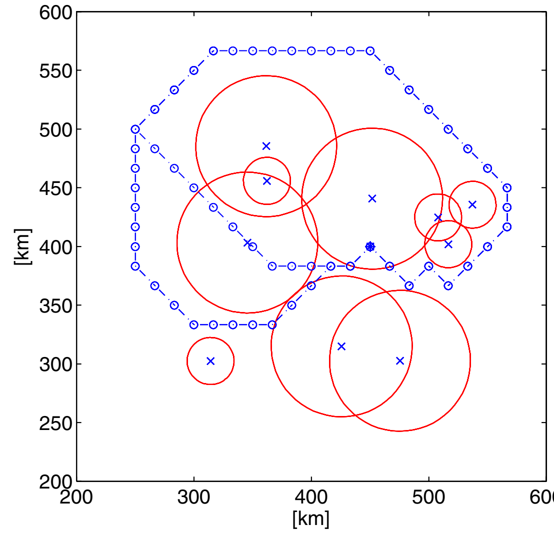

Fig. 5 Three flight paths with different fuel constraints.

visibility graphs [14], or a classical grid. This choice should be made depending on the environment and nature of each type of UAV mission, but because graph choice is not the focus of our work, we choose the simple grid depicted in Fig. 3. Note that this choice is made to illustrate the main concepts of this paper, not to give the best possible path. Obviously, paths restricted to eight main directions of travel are not tactically optimal.

To illustrate the standard way of UAV path planning, we first form a SPP in which the cost of each path segment is calculated as a linear combination of path length and the risks of Fig. 2. Applying a standard Dijkstra type of algorithm [1], we get the results in Fig. 4.

Figure 4 illustrates the lack of transparency offered to the operator. The difference between the paths starting at (250,520) and (250,530) is significant. From the former position, a southbound route is recommended that is short but fairly risky. From the latter position, a northbound route is suggested that is longer but safer. These differences should be controlled by operator preferences, not by small changes in starting position.

The objective of the path planning is to find the least risky path, respecting the given fuel (or time, see Remark 2) constraint. If there is a shortage of fuel and the UAV is at (250,530), the suggested northern route is not an option. And similarly, if there is a lot of fuel available and the UAV is at (250,520), the southern route is unnecessarily dangerous, and the northern route is the preferred option. Using the WCSPP instead of the SPP enables the operator to make this distinction directly, instead of having to tune the coefficients in the combination of path length and risk. Furthermore, using the RFPP formulation, the operator gets clear risk estimates for each path that enable him or her to make better tactical decisions.

The results of the RFPP are shown in Fig. 5. Paths for three different fuel bounds corresponding to three different maximal path lengths are shown. The path corresponding to a range of 300 km is forced to go straight through a threat area, barely avoiding the overlapping threat region, to reach the target. The corresponding overall mission risk is estimated to a survival probability of only 6%. A range of 400 km corresponds to the southernmost path with a survival probability of 22%, and a range of 600 km gives the northernmost path with a survival probability of 58%. These numbers are likely to make the operator abort the mission if there is only fuel for 300 km. Note again that the numbers are very coarse estimates, and when making the decision, the operators can often take information into account that is not available to an algorithm. However, a survival probability is much easier to interpret than, for example, a number representing overall mission radar exposure (see Remark 3). Finally, please note again that the purpose of the example is to illustrate the idea of risk-vs-fuel path planning, not to serve as an example of a good UAV mission path. For example, the zigzagging near (500,370) is tactically insane. A more complex choice of *G* must be used to plan real missions, perhaps in combination with some postprocessing of the path.

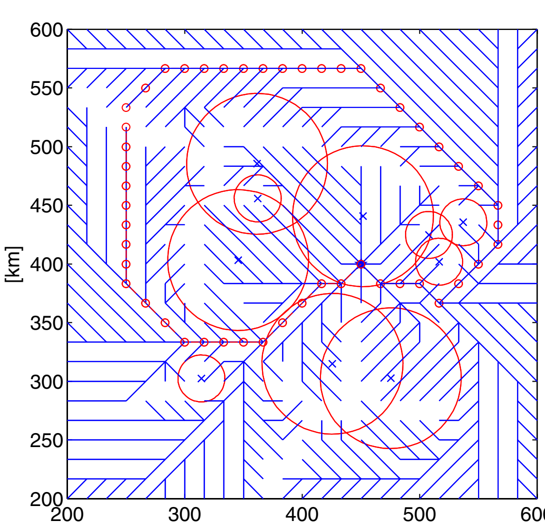

Fig. 4 Solution paths to a shortest-path problem using a weighted sum of distance and threat exposure.

V. Conclusions

In this paper, we have shown that it is possible to improve the transparency of UAV path planning in two ways. The first way is to include fuel or time constraints as actual optimization constraints instead of as parts of the objective function. The second way is to use risk estimates of individual path segments in the planning. The combination of both these suggestions enables the algorithm to maximize an estimate of the overall mission success rate, given the constraints imposed by the available amount of fuel, and to present this estimated rate to the UAV operator making tactical decisions.

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